

Gravitational Effects of Quantum Fields in the Interior of a Cylindrical Black Hole

Andrew DeBenedictis *

Department of Physics

Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6

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Abstract

The gravitational back-reaction is calculated for the conformally invariant scalar field within a black cosmic string interior with cosmological constant. Using the perturbed metric, the gravitational effects of the quantum field are calculated. It is found that the perturbations initially strengthen the singularity. This effect is similar to the case of spherical symmetry (without cosmological constant). This indicates that the behaviour of quantum effects may be universal and not dependent on the geometry of the spacetime nor the presence of a non-zero cosmological constant.

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1 Introduction

The gravitational effects of quantum fields in black hole spacetimes has long been studied. Since Hawking's discovery that black holes radiate [1] much interesting work has been done in this area. Quantities of interest include the expectation value $\langle \phi^2 \rangle$, which describe vacuum polarization effects,

*e-mail:adebened@sfu.ca

and $\langle T_{\mu\nu} \rangle$, the expectation value of the stress-energy tensor of the field. This latter quantity may then be used in the Einstein Field equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi\langle T_{\mu\nu} \rangle \quad (1)$$

to determine the back-reaction of the field on the original spacetime. The effects of the back-reaction may also include the removal of singularities ([2]-[7]). This is the main motivation for the work presented here and the answer would have consequences to many fundamental questions including the information loss problem.

Hiscock et.al [7] have done an extensive study of these effects on the Schwarzschild interior and have found cases where curvature is initially slowed in the interior as well as cases where the curvature is initially strengthened (such as the case of the massless conformally coupled scalar field). They have also studied the effects on black hole anisotropy. It is interesting to ask whether or not the results are a product of the symmetry chosen or are general. It is also interesting to ask whether the presence of a cosmological constant will alter the situation. The study here attempts to address both issues by studying a black cosmic string which is asymptotically anti-deSitter. The field is in the Hartle-Hawking vacuum state ¹ [11] and the stress-energy tensor is found using the approximation of Page [12] which is particularly useful here since the spacetime is an Einstein spacetime (in an Einstein spacetime the relation $R_{\mu\nu} = \Lambda g_{\mu\nu}$ holds). Black string solutions are of relevance to cosmic strings. It has also been shown how such black holes may form by gravitational collapse [13] [14]. This type of collapse has astrophysical relevance as the collapse of a finite spindle can behave as an infinite cylinder near its central region [15].

It may be thought that, since no external observer can view the interior without falling into the black hole, that a study of the interior is not physically meaningful. However, as pointed out in [7] black hole evaporation reveals more and more of the black hole interior as time progresses and therefore the interior has relevance to exterior observers in this way. Also, the issue of whether or not spacetime singularities actually exist has been one of intense interest ever since Oppenheimer and Snyder's [16] original collapse calculation.

¹Due to the fact that the black string has positive specific heat, the Hartle-Hawking vacuum state is particularly applicable here. For a discussion of black string thermodynamic properties see [8], [9] and [10].

2 Black String Spacetime

The black hole studied here is the cylindrical black hole spacetime developed by Lemos and Zanchin [17] and also studied by Kaloper [18]. If charge and angular momentum are not present the metric has the form

$$ds^2 = -(\alpha^2 \rho^2 - \frac{4M}{\alpha \rho}) dt^2 + \frac{d\rho^2}{(\alpha^2 \rho^2 - \frac{4M}{\alpha \rho})} + \rho^2 d\varphi^2 + \alpha^2 \rho^2 dz^2. \quad (2)$$

where M is the mass per unit length, $\alpha^2 = -\frac{1}{3}\Lambda$ and the coordinates take on the following ranges:

$$\begin{aligned} -\infty < t < \infty, \\ 0 \leq \rho < \infty, \\ 0 \leq \varphi < 2\pi, \\ -\infty < z < \infty. \end{aligned}$$

An event horizon exists at $\rho = \rho_H \equiv \frac{(4M)^{1/3}}{\alpha}$ and the cosmological constant (which is negative and necessary for cylindrical black hole solutions), Λ , dominates in the limit $\rho \rightarrow \infty$ giving the spacetime its asymptotically anti-deSitter behaviour.

The apparently singular behaviour of the spacetime at $\rho = \rho_H$ is a coordinate effect and not a true singularity. On calculating the Kretschmann scalar one obtains

$$K \equiv R_{\delta\lambda\mu\nu} R^{\delta\lambda\mu\nu} = 24\alpha^4 \left(1 + \frac{8M^2}{\alpha^6 \rho^6} \right) \quad (3)$$

from which it can be seen that the only true singularity is a polynomial singularity at $\rho = 0$. Thus, this solution violates the hoop conjecture but not the cosmic censorship conjecture. The hoop conjecture is therefore not valid in spacetimes with a cosmological constant.

As calculations will be extended to the interior, it is convenient to re-write the metric using the following coordinate redefinitions:

$$\begin{aligned} t &\rightarrow R, \\ \rho &\rightarrow T. \end{aligned}$$

Where T is timelike in the interior and R is spacelike. The “interior” metric now has the form

$$ds_{interior}^2 = -\frac{dT^2}{\left(\frac{4M}{\alpha T} - \alpha^2 T^2\right)} + \left(\frac{4M}{\alpha T} - \alpha^2 T^2\right) dR^2 + T^2 d\varphi^2 + \alpha^2 T^2 dz^2 \quad (4)$$

where the interior region corresponds to $0 \leq T \leq \rho_H$.

3 Stress-Energy Tensor

In this section the stress energy tensor is calculated which will eventually be used in (1) to calculate back-reaction effects. The expectation value of stress-energy tensors have been calculated in exterior Schwarzschild space-time by Howard and Candelas [19] and Page [12] as well as by Anderson et. al [20] who studied the stability in the extreme Reissner-Nordström black hole. Anderson, Hiscock and Samuel [21] have developed an approximation for both massive and massless fields in arbitrary spherically symmetric spacetimes and have used this approximation to calculate $\langle T_{\mu\nu} \rangle$ in the exterior Reissner-Nordström geometry. The Kerr and Kerr-Newman spacetimes have also been studied in [22], [23] and [24]. Quantum effects in lower dimensional black hole exteriors may be found in [25]-[34]. For a calculation of $\langle \phi^2 \rangle$ in the spacetime studied here see [35].

Various works on back-reaction effects of quantum fields have also been produced. Hiscock and Weems [36], Bardeen [37], Balbinot [38] and York [39] have studied effects in Schwarzschild and Reissner-Nordström exteriors. Few calculations, however, have been performed on the interiors of black holes. One such study has been done by Hiscock, Larson and Anderson [7] where they have extended their analysis to the Schwarzschild interior and calculated back-reaction effects on curvature invariants.

3.1 Stress-Energy Tensor for the Conformally Coupled Scalar Field

The calculation of the stress energy tensor will be done using the Euclideanized metric. This is obtained by making the transformation ($t \rightarrow -i\tau$) in (2) giving the metric positive definite signature so that

$$ds_{Euclidean}^2 = (\alpha^2 \rho^2 - \frac{4M}{\alpha \rho}) d\tau^2 + \frac{d\rho^2}{(\alpha^2 \rho^2 - \frac{4M}{\alpha \rho})} + \rho^2 d\varphi^2 + \alpha^2 \rho^2 dz^2. \quad (5)$$

To calculate $\langle T_{\mu\nu} \rangle$ exactly is an extremely difficult task which normally involves acting on $\langle \phi^2 \rangle$ with a complicated differential operator. It is useful therefore to use an approximation which will give an analytic result from which information on back-reaction effects may be calculated. The approximation used here is the approximation of Page for thermal stress-energy tensors in static spacetimes[12]. This approximation is especially good if the spacetime under consideration is an Einstein spacetime such as the one considered here and contains no ambiguities in the case of scalar fields. The Bekenstein-Parker [40] Gaussian path integral approximation is utilized from which the thermal propagator is constructed. This construction is done in an (Euclideanized) ultrastatic spacetime ($g_{00} = k$, k is a constant chosen to be 1 in this work) which is related to the physical spacetime by

$$g_{\mu\nu} = |g_{00(p)}|^{-1} g_{\mu\nu(p)}. \quad (6)$$

The subscript p will be used to indicate quantities calculated using the physical metric (all other tensors in this section are obtained using the ultrastatic metric). This approximation gives, for the stress-energy tensor in the physical spacetime:

$$T_{\nu(p)}^{\mu} = |g_{00(p)}|^{-2} \{ T_{\nu}^{\mu} + [8\lambda |g_{00(p)}|^{-1} (|g_{00(p)}|^{1/2})_{;\alpha} (|g_{00(p)}|^{1/2})^{;\beta} - 4(\lambda + \beta) R_{\alpha}^{\beta} C_{\beta\nu}^{\alpha\mu} + 2\beta [H_{\nu}^{\mu} + 3\alpha^4 |g_{00(p)}|^2 \delta_{\nu}^{\mu}] + \frac{1}{6} \gamma I_{\nu}^{\mu} \}, \quad (7)$$

where $C_{\beta\nu}^{\alpha\mu}$ is the Weyl tensor and the coefficients λ , β and γ are given as follows:

$$\lambda = \frac{12h(0)}{2^9 45 \pi^2}, \quad \beta = \frac{-4h(0)}{2^9 45 \pi^2}, \quad \gamma = \frac{8h(0)}{2^9 45 \pi^2}. \quad (8)$$

The number of helicity states, $h(0)$, simply counts the number of scalar fields present. T_{ν}^{μ} is the stress-energy tensor in the ultrastatic metric,

$$T_{\nu}^{\mu} = \frac{\pi^2}{90} T^4 (\delta_{\nu}^{\mu} - 4\delta_0^{\mu} \delta_{\nu}^0), \quad (9)$$

where T is the temperature of the black string which can be found by demanding that the Euclidean extension of (2) be regular on the horizon;

$$T = \frac{3\alpha}{4\pi} (4M)^{1/3}. \quad (10)$$

The quantities H_ν^μ and I_ν^μ are given by:

$$\begin{aligned} H_\nu^\mu &= -R^{\alpha\mu}R_{\alpha\nu} + \frac{2}{3}RR_\nu^\mu + \left(\frac{1}{2}R_\beta^\alpha R_\alpha^\beta - \frac{1}{4}R^2\right)\delta_\nu^\mu, \\ I_\nu^\mu &= 2R_{;\nu}^\mu - 2RR_\nu^\mu + \left(\frac{1}{2}R^2 - 2R_{;\alpha}^\alpha\right)\delta_\nu^\mu. \end{aligned} \quad (11)$$

The calculation of $\langle T_{\mu\nu} \rangle$ is carried out on the exterior of the black hole. However, since the result is finite at the horizon, it is easily extended to the interior where the field equations will be solved. For the spacetime considered here, the stress-energy tensor is calculated to be

$$\begin{aligned} T_{\nu(p)}^\mu &= -\{1920\pi^2\epsilon[\alpha^4\rho^6(\alpha^3\rho^3 - 4M)^2]\}^{-1}[-27\,2^{2/3}\,M^{4/3}\alpha^8\rho^8(\delta_\nu^\mu - 4\delta_0^\mu\delta_\nu^0) \\ &\quad -16M\alpha^9\rho^9(3\delta_0^\mu\delta_\nu^0 + \delta_1^\mu\delta_\nu^1) - 128\alpha^3\rho^3M^3(\delta_\nu^\mu - 12\delta_0^\mu\delta_\nu^0) \\ &\quad +192M^4(\delta_\nu^\mu - 12\delta_0^\mu\delta_\nu^0 - \frac{8}{3}\delta_1^\mu\delta_\nu^1) \\ &\quad +96M^2\alpha^6\rho^6(\delta_\nu^\mu - 5\delta_0^\mu\delta_\nu^0 + \delta_1^\mu\delta_\nu^1) + 2\alpha^{12}\rho^{12}\delta_\nu^\mu], \end{aligned} \quad (12)$$

where $\epsilon = \hbar\alpha^2$. This function remains unchanged when analytically continued to the Lorentzian sector by the transformation $\tau \rightarrow it$ and has trace consistent with anomaly calculations. Far from the black string, (12) takes on its pure anti-deSitter value of $-\frac{\alpha^4}{960\pi^2}\delta_\nu^\mu$ [41] whereas at the horizon (12) is also well defined and given by

$$T_{\nu(p)}^\mu(\rho_H) = \frac{\alpha^4}{\pi^2} \begin{pmatrix} \frac{1}{640} & 0 & 0 & 0 \\ 0 & \frac{1}{640} & 0 & 0 \\ 0 & 0 & -\frac{1}{640} & 0 \\ 0 & 0 & 0 & -\frac{1}{640} \end{pmatrix}. \quad (13)$$

Inspection of (13) immediately shows that the weak energy condition (WEC) is violated. The qualitative behaviour of the energy density ($\varepsilon = -T_0^0$) is shown in figure 1 where it can be seen that the WEC is violated throughout the interior of the black hole ($\rho < \sim 1.6$). However, it is unknown how relevant the classical energy conditions are in the case of quantum matter where violations are common (for example in the case of the Casimir effect) and are in fact required for a self consistent picture of Hawking evaporation.

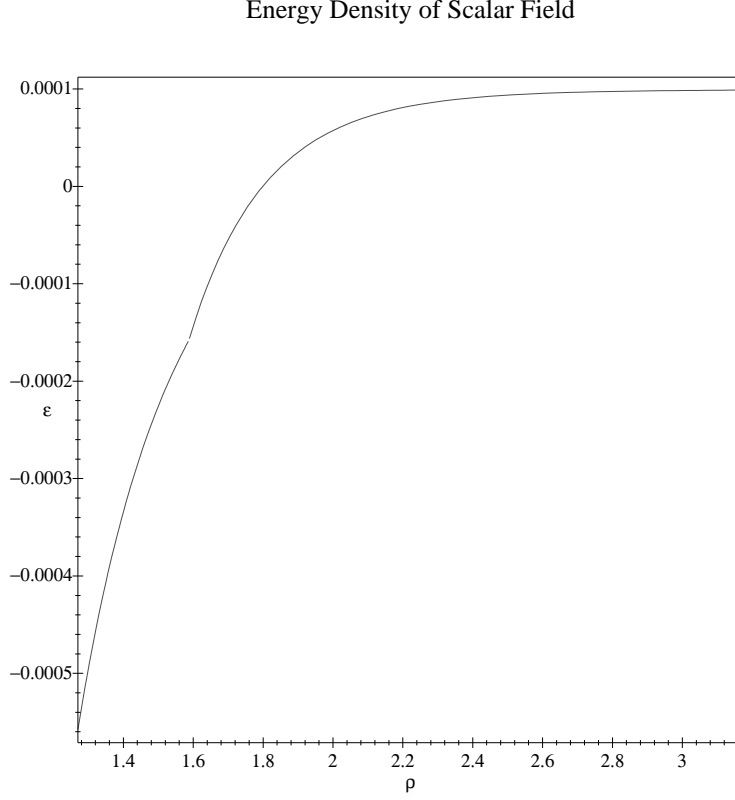


Figure 1: Energy density of the quantum scalar field in the cylindrical black hole spacetime. Weak energy condition violation can be seen throughout the interior ($\rho < \sim 1.6$) and part of the exterior. The interior energy density is given by $-T_T^T$ whereas on the exterior it is given by $-T_t^t$.

4 Gravitational Back-Reaction

In this section the gravitational effects of the quantum field on the background spacetime will be calculated using the perturbed metric

$$ds^2 = -\frac{dT^2}{\frac{4M}{\alpha T} - \alpha^2 T^2} (1 + \epsilon\eta(T)) + \left(\frac{4M}{\alpha T} - \alpha^2 T^2 \right) (1 + \epsilon\sigma(T)) dR^2 + T^2 d\varphi^2 + \alpha^2 T^2 (1 + \epsilon\psi(T)) dz^2. \quad (14)$$

The functions $\eta(T)$, $\sigma(T)$ and $\psi(T)$ are to be solved for and the coupling constant, ϵ is assumed to be small. The Einstein field equations, to first

order in ϵ yield

$$\sigma(T)'(\alpha^3 T^3 - 4M) + \psi(T)'(\alpha^3 T^3 + M) = \left(\frac{8\pi \langle T_T^T \rangle}{\epsilon} + 3\alpha^2 \eta(T) \right) \alpha T^2, \quad (15a)$$

$$\begin{aligned} & \frac{d}{dT} \left[\eta(T)(4M - \alpha^3 T^3) + \psi(T)' \left(\frac{\alpha^3 T^4}{2} - 2TM \right) \right] - 3M\psi(T)' \\ &= \frac{8\pi \langle T_R^R \rangle \alpha T^2}{\epsilon}, \end{aligned} \quad (15b)$$

$$\begin{aligned} & \frac{d}{dT} \left[(\sigma(T)' + \psi(t)') \left(\frac{\alpha^3 T^4}{2} - 2TM \right) \right] + 3M\sigma(T)' + \eta(T)'(M - \alpha^3 T^3) \\ &= \left(\frac{8\pi \langle T_\varphi^\varphi \rangle}{\epsilon} + 3\alpha^2 \eta(T) \right) \alpha T^2, \end{aligned} \quad (15c)$$

$$\begin{aligned} & \frac{d}{dT} \left[\sigma(T)' \left(\frac{\alpha^3 T^4}{2} - 2TM \right) \right] + 3M\sigma(T)' + \eta(T)'(M - \alpha^3 T^3) \\ &= \left(\frac{8\pi \langle T_z^z \rangle}{\epsilon} + 3\alpha^2 \eta(T) \right) \alpha T^2, \end{aligned} \quad (15d)$$

where primes denote ordinary differentiation with respect to T .

Since solving (15a-15d) using (12) and calculating the resulting relevant quantities such as the Riemann curvature tensor and Kretschmann scalar would be an enormous task, some simplifying assumptions are first made. It is noted from (12) that equations (15c) and (15d) must be equal. It is therefore assumed that the function $\psi(T)$ is equal to a constant and therefore does not appear in the field equations. Secondly, the stress energy tensor will be approximated by its value near the event horizon. This should not introduce too large of an error in the calculations as the perturbative scheme here is only valid in regions where the spacetime curvature is not large (such as near the horizon in the small α limit).

Utilizing the above, the following solutions are obtained for the per-

turbations:

$$\eta(T) = \frac{1}{\pi(4M - \alpha^3 T^3)} \left[\frac{13}{240} \alpha^3 T^3 - \frac{3}{160} \alpha^4 T^4 M^{-1/3} - \frac{M}{15} \right] \quad (16)$$

$$\begin{aligned} \sigma(T) &= -\frac{1}{\pi(4M - \alpha^3 T^3)} \left[\frac{13}{240} \alpha^3 T^3 - \frac{3}{160} \alpha^4 T^4 M^{-1/3} - \frac{M}{15} \right] \\ &+ \frac{1}{30} \frac{\alpha T 2^{1/3}}{\pi M^{1/3}} + \frac{1}{45\pi} \ln \left(\frac{(4M)^{1/3} - \alpha T}{4M - \alpha^3 T^3} \right) \\ &- \frac{1}{90\pi} \ln(\alpha^2 T^2 + \alpha T (4M)^{1/3} + (4M)^{2/3}) \\ &- \frac{\sqrt{3}}{45\pi} \arctan \left[\frac{1}{\sqrt{3}} \left(\frac{\alpha T 2^{1/3}}{M^{1/3}} + 1 \right) \right] + k_0. \end{aligned} \quad (17)$$

The integration constant k_0 may be left arbitrary as it does not enter subsequent calculations. Both solutions are well behaved in the domain of validity as $\eta(\rho_H) = \frac{1}{240\pi}$ and $\sigma(\rho_H) = \frac{153 - 72 \ln(3) - 96 \ln(2) + 48 \ln(M) + 16\sqrt{3}\pi}{2160\pi}$.

Attention is now turned to the effects of the quantum perturbation on the black hole spacetime. It has long been thought that quantum effects may remove the singular behaviour of physical spacetimes. Although the perturbative scheme can not determine whether or not the actual singularity is removed, it can give information on the growth of curvature scalars on the interior spacetime. Using the perturbed metric, the Kretschmann scalar can now be written as:

$$K = K_{orig} + \epsilon \delta K, \quad (18)$$

where K_{orig} is the unperturbed value given by (3) and δK is the first order correction term. Whether or not curvature is strengthened depends on the sign of δK . If it is positive, the initial curvature growth is strengthened. If it is negative, it is weakened. The analysis will be limited to the region near the horizon.

The correction term is calculated to be

$$\begin{aligned}
\delta K = & \frac{-1}{15} [25 \alpha^{17} T^{17} 2^{2/3} + 54 \alpha^{16} T^{16} (2M)^{1/3} + 116 \alpha^{15} T^{15} M^{2/3} \\
& - 286 \alpha^{14} T^{14} 2^{2/3} M - 624 \alpha^{13} T^{13} 2^{1/3} M^{4/3} - 1352 \alpha^{12} T^{12} M^{5/3} \\
& + 1104 \alpha^{11} T^{11} 2^{2/3} M^2 + 2256 \alpha^{10} T^{10} 2^{1/3} M^{7/3} 4608 \alpha^9 T^9 M^{8/3} \\
& - 2656 \alpha^8 T^8 2^{2/3} M^3 - 3456 \alpha^7 T^7 2^{1/3} M^{10/3} - 3200 \alpha^6 T^6 M^{11/3} \\
& + 9472 \alpha^5 T^5 2^{2/3} M^4 + 9984 \alpha^4 T^4 2^{1/3} M^{13/3} + 2048 \alpha^3 T^3 M^{14/3} \\
& - 18432 \alpha^2 T^2 2^{2/3} M^5 - 24576 \alpha T 2^{1/3} M^{16/3} - 24576 M^{17/3}] \\
& \times [\alpha^2 T^6 M^{1/3} \pi (\alpha^2 T^2 + \alpha T (4M)^{1/3} + (4M)^{2/3})^2 \\
& \times (4M^{2/3} + \alpha^2 T^2 2^{2/3} + 2\alpha T (2M)^{1/3})^2 (\alpha T - (4M)^{1/3})^2]^{-1} \quad (19)
\end{aligned}$$

Although this result is slightly complicated, a few relevant properties may be obtained. The value of δK near the horizon behaves as

$$\delta K \approx \frac{3}{10} \frac{\alpha^4}{\pi} - 2 \frac{\alpha^5}{\pi} \left(\frac{2}{M} \right)^{1/3} \left(T - \frac{(4M)^{1/3}}{\alpha} \right), \quad (20)$$

so that the limiting value at $T = \rho_H$ is given by $\delta K(\rho_H) = \frac{3}{10} \frac{\alpha^4}{\pi}$. from which it can be seen that curvature growth is *strengthened* at the horizon. The function (20) increases as one passes through the horizon towards the singularity and the perturbation eventually becomes positive. Near the singularity the curvature diverges as $1/T^{16}$ although in this regime the approximation breaks down and the expression has no physical meaning.

For the case of Schwarzschild geometry the curvature perturbation diverges as $1/T^9$ near the singularity for massless conformally coupled scalars [7] and in the regime where the perturbation is valid, curvature invariants are always strengthened. At the event horizon of a Schwarzschild black hole, for example, the perturbation is $\delta K = \frac{1965}{2M^4 45 \times 2^{13} \pi}$.

5 Conclusion

The stress energy tensor for a conformally coupled quantum scalar field has been calculated in the black string spacetime and it is found that, as is common with quantum fields in curved spacetime, there exist regions where the weak energy condition is violated. The violation occurs on the interior and near the horizon on the exterior of the black hole. From the stress energy

tensor, the back-reaction has been calculated in the form of the perturbed metric and Kretschmann scalar. Similar to the case of spherical symmetry without cosmological constant, it is found that curvature is strengthened on the interior indicating that quantum effects may be geometry and cosmological constant independent.

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